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Using SU(3) Relations to bound the CP Asymmetries in $B \to KKK$ decays

Guy Engelhard* and Guy Raz†

Department of Particle Physics, Weizmann Institute of Science, Rehovot 76100, Israel

Abstract

We consider three body $\Delta s = 1$ $B \to f$ decays with $f = KKK$. The deviations of $-\eta_f S_f$ from $S_{\psi K_S}$ and of C_f from zero can be bounded using the approximate $SU(3)$ flavor symmetry of the strong interactions and branching ratios of various $\Delta s = 0$ modes. We present the most promising $SU(3)$ amplitude relations that can be used to obtain these bounds.

*Electronic address: guy.engelhard@weizmann.ac.il

†Electronic address: guy.raz@weizmann.ac.il

I. INTRODUCTION

Recently there has been a growing interest, driven by experiments [1, 2], in the CP asymmetries of $b \to s$ penguin dominated processes. For final CP eigenstates the CP asymmetry is time dependent:

$$\mathcal{A}_f(t) = \frac{\Gamma(\overline{B}^0(t) \to f) - \Gamma(B^0(t) \to f)}{\Gamma(\overline{B}^0(t) \to f) + \Gamma(B^0(t) \to f)} = -C_f \cos(\Delta m_B\, t) + S_f \sin(\Delta m_B\, t). \tag{1}$$

For flavor specific states, the CP asymmetry is time independent:

$$\mathcal{A}_f(t) = \mathcal{A}_f. \tag{2}$$

Within the SM, these processes are dominated by a single phase, while a second phase is CKM suppressed by $O(\lambda^2)$. Consequently, within the SM, $-\eta_f S_f \approx S_{\psi K_S}$ and $C_f, \mathcal{A}_f \approx 0$. Given the present central values and errors of the experimental measurements [1, 2], it is quite possible that new physics is inducing shifts of $\mathcal{O}(0.1 - 0.4)$ from the SM value. To understand whether new physics is indeed involved, it is necessary to estimate the deviations allowed within the SM of the various $-\eta_f S_f$'s from $S_{\psi K_S}$. Various methods have been devised in order to estimate the deviations [3, 4, 5, 6, 7, 8]. These method suffer from hadronic uncertainties and, furthermore, are generally not suitable for use in three-body decays.

Recently, methods employing the $SU(3)$ symmetry to place hadronic-model independent bounds on CP asymmetries in $b \to s$ processes [9, 10, 11, 12, 13, 14] have been extended to three-body decay modes [15]. The final state studied in [15], $K_S K_S K_S$, is symmetric under the exchange of any two of the final particles. This situation allowed a considerable simplification of the $SU(3)$ analysis. In this work we extend the analysis to all other KKK modes. Here this exchange symmetry does not apply. A related study using factorization to estimate the asymmetry in some of these modes is presented in [16].

While the bounds we obtain are indeed hadronic-model independent, they do suffer from two limitations. First, the $SU(3)$ breaking is estimated to be of $\mathcal{O}(0.3)$. The bounds we obtain may be violated at this order. Second, since the $SU(3)$ analysis cannot infer the phases of physical amplitudes, we conservatively add the amplitudes coherently. Generically, this means that our upper bounds can be considerably weaker than the actual values.

The experimental data concerning CP asymmetries in $B \to KKK$ decays is summarized in table I. The result of $S_{K^+K^-K_{S,L}}$ takes into account both $S_{K^+K^-K_S}$ and $S_{K^+K^-K_L}$. The CP

Mode	$-\eta_f S_f$	$C_f,\, -\mathcal{A}_f$	Ref.
$K_S K_S K_S$	0.26 ± 0.34	-0.41 ± 0.21	[17]
$K^+ K^- K_{S,L}$	0.51 ± 0.13	0.08 ± 0.09	[1]
$K_S K_S K_L$	—	—	
$K_S K_S K^\pm$	n/a	0.04 ± 0.11	[17]
$K^\pm K^+ K^-$	n/a	-0.02 ± 0.08	[17]
$K_S K_L K^\pm$	n/a	—	

TABLE I: Measured CP asymmetries in $B \to KKK$ decays.

asymmetries of $B \to K_S K_S K_L$ and $B^\pm \to K_S K_L K^\pm$ are not yet measured. For completeness we include here also the result for $B \to K_S K_S K_S$ which was studied in [15].

The plan of the paper is as follows. In section II we review the notations and formalism relevant to three body decays. In section III we list the amplitude relations for $B \to KKK$ decays. We conclude in section IV. We quote the experimental branching ratios we use in appendix A.

II. NOTATIONS AND FORMALISM

In this section we review the notations and formalism relevant to 3-body decays introduced in [15] (where a much more detailed discussion is presented). We use abstract vector notation, *e.g.* $\vec{A}_f$, where the vector index runs over all possible values for the quantum numbers, to describe the various states. The total decay rate is given by

$$\Gamma(B^0 \to f) = \left\| \vec{A}_f \right\|^2 . \tag{3}$$

Experiments measure the averaged rates given by:

$$\Gamma(B \to f) = \frac{1}{2}\left[\Gamma(B^0 \to f) + \Gamma(\overline{B}^0 \to \overline{f})\right], \tag{4}$$

where $\overline{f}$ is the CP-conjugate state of f.

The norm on the right hand side of (3) represents a sum over all possible final states, that is, all momentum configurations. In order to derive $SU(3)$ relations, we choose to span the final states in a basis with definite linear momenta. Our convention is that the order in

which we write the three final mesons corresponds to their momentum configuration:

$$|M_i M_j M_k\rangle \equiv |M_i(p_1)M_j(p_2)M_k(p_3)\rangle \ . \tag{5}$$

We write a generic $\Delta s = 1$, $B^0 \to f$ decay amplitude as follows:

$$\vec{A}_f = V_{cb}^* V_{cs}\, \vec{a}_f^c + V_{ub}^* V_{us}\, \vec{a}_f^u . \tag{6}$$

Here, and for all other processes discussed below, the amplitudes for the CP-conjugate processes, $\overline{B}^0 \to \overline{f}$, have the CKM factors complex-conjugated, while the $\vec{a}_f^{u,c}$ factors remain the same.

We define two parameters:

$$\xi_f \equiv \ \frac{|V_{ub}^* V_{us}|}{|V_{cb}^* V_{cs}|} \frac{\vec{a}_f^c \cdot \vec{a}_f^u}{\|\vec{a}_f^c\|^2} \ , \tag{7}$$

$$\left|\overline{\xi}_f\right| \equiv \ \frac{|V_{ub}^* V_{us}|}{|V_{cb}^* V_{cs}|} \frac{\|\vec{a}_f^u\|}{\|\vec{a}_f^c\|} \ , \tag{8}$$

where

$$\frac{|\xi_f|}{\left|\overline{\xi}_f\right|} \leq 1 \ . \tag{9}$$

The parameter $\left|\overline{\xi}_f\right|$ is the one which can be constrained by $SU(3)$ relations, and that leads, through eq. (9), to a constraint on $|\xi_f|$. The CP asymmetries can be written to first order in $\mathcal{R}e(\xi_f)$ and $\mathcal{I}m(\xi_f)$ as follows:

$$-\eta_f S_f - S_{\psi K_S} = 2\, \cos 2\beta\, \sin\gamma\, \mathcal{R}e(\xi_f) \ , \tag{10}$$

$$-\mathcal{A}_f,\, C_f = -2\, \sin\gamma\, \mathcal{I}m(\xi_f) \ . \tag{11}$$

We write a generic $\Delta s = 0$ decay amplitude as follows:

$$\vec{A}_f = V_{cb}^* V_{cd}\, \vec{b}_f^c + V_{ub}^* V_{ud}\, \vec{b}_f^u \ . \tag{12}$$

$SU(3)$ symmetry leads to amplitude relations of the form

$$\vec{a}_f^q = \sum_{f'} X'_{f'}\, \vec{b}_{f'}^q \qquad (q = u \text{ or } c). \tag{13}$$

Taking the norm of eq. (13) needs to be done with care: the sum can involve states with different symmetry properties under exchange of final particles since the $\Delta s = 1$ mode and the $\Delta s = 0$ modes may have different symmetries with respect to the momentum variables

(so that integration over phase space is different for the various modes). Taking the norm of both sides, the left hand side can be bounded from above as:

$$\left\|\vec{a}_f^q\right\| \leq \sum_{f'} |X_{f'}| \left\|\vec{b}_{f'}^q\right\| \ , \tag{14}$$

where $X_{f'}$ and $X'_{f'}$ are related by symmetry factors.

In order to bound $\left|\overline{\xi}_f\right|$ with no additional assumptions [9, 15], we define another parameter, $\left|\widehat{\overline{\xi}}_f\right|$,

$$\left|\widehat{\overline{\xi}}_f\right|^2 \equiv \left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{\left\|V_{cb}^* V_{cd}\,\vec{a}_f^c + V_{ub}^* V_{ud}\,\vec{a}_f^u\right\|^2 + \left\|V_{cb} V_{cd}^*\,\vec{a}_f^c + V_{ub} V_{ud}^*\,\vec{a}_f^u\right\|^2}{\left\|V_{cb}^* V_{cs}\,\vec{a}_f^c + V_{ub}^* V_{us}\,\vec{a}_f^u\right\|^2 + \left\|V_{cb} V_{cs}^*\,\vec{a}_f^c + V_{ub} V_{us}^*\,\vec{a}_f^u\right\|^2} \ . \tag{15}$$

The numerator and denominator of $\left|\widehat{\overline{\xi}}_f\right|^2$ are related to charge-averaged rates:

$$\left\|V_{cb}^* V_{cd}\,\vec{a}_f^c + V_{ub}^* V_{ud}\,\vec{a}_f^u\right\|^2 + \left\|V_{cb} V_{cd}^*\,\vec{a}_f^c + V_{ub} V_{ud}^*\,\vec{a}_f^u\right\|^2 \leq 2\left(\sum_{f'} |X_{f'}| \sqrt{\Gamma(B \to f')}\right)^2 \ , \tag{16}$$

$$\left\|V_{cb}^* V_{cs}\,\vec{a}_f^c + V_{ub}^* V_{us}\,\vec{a}_f^u\right\|^2 + \left\|V_{cb} V_{cs}^*\,\vec{a}_f^c + V_{ub} V_{us}^*\,\vec{a}_f^u\right\|^2 = 2\Gamma(B \to f) \ . \tag{17}$$

Using the measured charge-averaged rates, a constraint on $\left|\widehat{\overline{\xi}}_f\right|^2$ is obtained.

The $\left|\widehat{\overline{\xi}}_f\right|$ and $\left|\overline{\xi}_f\right|$ parameters are related as follows:

$$\left|\widehat{\overline{\xi}}_f\right|^2 = \frac{\left|\frac{V_{us}V_{cd}}{V_{cs}V_{ud}}\right|^2 + \left|\overline{\xi}_f\right|^2 + 2\cos\gamma\,\mathcal{R}e\left(\frac{V_{us}V_{cd}}{V_{cs}V_{ud}}\,\xi_f\right)}{1 + \left|\overline{\xi}_f\right|^2 + 2\cos\gamma\,\mathcal{R}e(\xi_f)} \ . \tag{18}$$

The relation (18) has the property that for $\lambda^2 \lesssim \left|\widehat{\overline{\xi}}_f\right| \leq 1$ we get a constraint on $\left|\overline{\xi}_f\right|$, for any ξ_f (of course, within the allowed range, $|\xi_f| \leq \left|\overline{\xi}_f\right|$, see eq. (9)). Since we do not know the value of ξ_f, we should consider the weakest constraint, which corresponds to $\mathcal{R}e(\xi_f) = \left|\overline{\xi}_f\right|$ (the $(V_{us}V_{cd})/(V_{cs}V_{ud})$ term is experimentally known to be real to a good approximation). The weakest bound, which corresponds to $\mathcal{R}e(\xi_f) = \left|\overline{\xi}_f\right|$ and $\gamma = 0$, is obtained from the curve $\left|\widehat{\overline{\xi}}_f\right| = (\left|\overline{\xi}_f\right| - \lambda^2)/(1 + \left|\overline{\xi}_f\right|)$.

In the following sections, we present the $SU(3)$ analysis for the $B \to KKK$ modes. Before presenting the relations, however, we make two comments.

The first comment is related to the removal of resonances. In general, the cleanest result would be obtained with all resonances removed both from the CP asymmetries and from the branching ratios. Since, however, we use $SU(3)$ symmetry here, a resonance from a complete $SU(3)$ representation, which enters all relevant modes, would not harm the analysis as long as

the same strategy regarding it is employed in all branching ratios and in the CP asymmetry. One should also note that if resonances are small in the $B \to KKK$ decays we consider (see for example [18]), a removal of the resonances from the $\Delta s = 0$ branching ratio would only create a small error. On the other hand, if the resonances are removed from the branching ratio of the $B \to KKK$ mode, a failure to remove the resonances in the $\Delta s = 0$ modes would only weaken the bound but not invalidate it.

The second comment addresses our methodology of presenting the results. In order to find the $SU(3)$ relations of various $\Delta s = 1$ modes we follow the method outlined in [15]. We scan over all possible contractions of the relevant $SU(3)$ tensors, avoiding the need to discuss $SU(3)$ properties of tensor products. In contrast to [15], however, here we cannot use symmetrized states only, a fact that complicates the analysis in the following ways:

1. We have 92 $\Delta s = 0$ modes (including all possible permutations of the mesons) and 40 reduced matrix elements. This produces a table which is too large to include here.

2. Since there are 92 modes, presenting the most general relations with free parameters (see e.g. [9]) is not practical.

Since the contributions are added coherently, our constraints are weakened if too many modes are included. Consequently the fewer the modes involved in an $SU(3)$ relation, the better the chance of getting a strong bound. In the following section, we therefore present only the relations involving the smallest number of modes, hoping that (once measured) these will provide us with a strong bound on the deviations.

III. $SU(3)$ RELATIONS FOR $B \to KKK$ DECAYS

In this section we discuss the most promising $SU(3)$ relations relevant to each $B \to KKK$ mode. One can divide the KKK states into six distinct types which are determined by the identity of the K's and the symmetry of the states. States which are of the same type have the same ξ and related CP asymmetries. They are related to each other by K mixing factors. There are three types which are common states of B^0 and $\overline{B}^0$: $K_S K_S K_S$ ($K_L K_L K_L$), $K^+ K^- K_S$ ($K^+ K^- K_L$) and $K_S K_S K_L$ ($K_L K_L K_S$). The other three types are charged and therefore flavour specific: $K_S K_S K^\pm$ ($K_L K_L K^\pm$), $K^\pm K^+ K^-$ and $K_S K_L K^\pm$.

The first type was considered in [15], and the others are considered below.

A. $B \to K^+K^-K_{S,L}$

While the mode $K^+K^-K_{S(L)}$ is, strictly speaking, not a CP eigenstate, isospin [19] and angular momentum analysis [20] show that it is predominantly CP even (odd).

Note that the $\Delta s = 1$ state $K^+K^-K^0$ is related to the state $K^+K^-K_{S,L}$ through K mixing. We evaluate $|\xi_{K^+K^-K^0}|$, but from eq. (7) it is clear that $|\xi_{K^+K^-K^0}| = \left|\xi_{K^+K^-K_{S,L}}\right|$, where the latter is the quantity that enters the bounds in eqs. (10) and (11).

1. *Three $\Delta s = 0$ Amplitudes*

We find two amplitude relations involving three $\Delta s = 0$ modes:

$$\vec{a}^{\,q}_{K^+K^-K^0} = \sqrt{\frac{3}{2}}\,\vec{b}^{\,q}_{K^+K^-\eta_8} - \frac{1}{\sqrt{2}}\,\vec{b}^{\,q}_{K^+K^-\pi^0} + \vec{b}^{\,q}_{\pi^+K^-K^0}\ , \tag{19}$$

$$\vec{a}^{\,q}_{K^+K^-K^0} = \sqrt{\frac{3}{2}}\,\vec{b}^{\,q}_{\pi^+\pi^-\eta_8} - \frac{1}{\sqrt{2}}\,\vec{b}^{\,q}_{\pi^+\pi^-\pi^0} - \vec{b}^{\,q}_{K^+\pi^-\overline{K}^0}\ . \tag{20}$$

Only the second relation has all modes measured. (In the $SU(3)$ limit $\eta_8 = \eta$. However, if the corresponding modes involving η' are also measured, the $\eta - \eta'$ mixing can be taken into account explicitly. See [15] for more details.) Using this relation we can place a bound (experimental values for the branching ratios [17, 21] are presented in appendix A):

$$\left|\widehat{\overline{\xi}}_{K^+K^-K^0}\right| \leq 0.22\left(\sqrt{\frac{\frac{3}{2}\mathcal{B}(\pi^+\pi^-\eta_8)}{\mathcal{B}(K^+K^-K^0)}} + \sqrt{\frac{\frac{1}{2}\mathcal{B}(\pi^+\pi^-\pi^0)}{\mathcal{B}(K^+K^-K^0)}} + \sqrt{\frac{\mathcal{B}(K^+\pi^-\overline{K}^0)}{\mathcal{B}(K^+K^-K^0)}}\right) \leq 0.41. \tag{21}$$

This implies that

$$\left|\overline{\xi}_{K^+K^-K^0}\right| \leq 0.78. \tag{22}$$

Clearly better measurements are needed in order to make this bound more constraining.

2. *Dynamical Assumptions*

One can use simplifying dynamical assumptions by neglecting the effect of small contributions from exchange, annihilation, and penguin annihilation diagrams [22]. Such a simplification leads to new relations. We present relations involving two $\Delta s = 0$ modes:

$$\vec{a}^{\,q}_{K^+K^-K^0} = -\sqrt{2}\,\vec{b}^{\,q}_{K^+K^-\pi^0} + \vec{b}^{\,q}_{\pi^+K^-K^0}\ , \tag{23}$$

$$\vec{a}^{\,q}_{K^+K^-K^0} = \sqrt{6}\,\vec{b}^{\,q}_{K^+K^-\eta_8} + \vec{b}^{\,q}_{\pi^+K^-K^0}\ . \tag{24}$$

Upper bounds for the branching ratios of the modes in (23) have been obtained. Using these we get

$$\left|\widehat{\overline{\xi}}_{K^+K^-K^0}\right| \leq 0.22\left(\sqrt{\frac{2\mathcal{B}(K^+K^-\pi^0)}{\mathcal{B}(K^+K^-K^0)}}+\sqrt{\frac{\mathcal{B}(\pi^+K^-K^0)}{\mathcal{B}(K^+K^-K^0)}}\right) \leq 0.48. \tag{25}$$

This implies that

$$\left|\overline{\xi}_{K^+K^-K^0}\right| \leq 1.02, \tag{26}$$

a weaker constraint compared to eq. (22). Again, better measurements may improve this constraint.

B. $B \to K_S K_S K_L$ ($K_L K_L K_S$)

The $K_S K_S K_L$ mode, although having the same $\Delta s = 1$ contribution as $K_S K_S K_S$ (namely, $K^0 K^0 \overline{K}^0$) does not necessarily have the same $SU(3)$ relations. This is due to the fact that the $SU(3)$ relations given in [15] depend only on the symmetric part of the amplitude. The full amplitude fulfills only a subset of the $SU(3)$ relations given there.

Since the mode $K^0 K^0 \overline{K}^0$ is symmeric under the exchange of two of its constituents, the strongest $SU(3)$ bounds come from modes having that symmetry as well. We define

$$|\mathcal{S}(M_1M_2)M_3\rangle = \frac{1}{\sqrt{2}}(|M_1M_2M_3\rangle + |M_2M_1M_3\rangle)\ , \tag{27}$$

$$|\mathcal{S}(M_1M_1)M_2\rangle = |M_1M_1M_2\rangle\ , \tag{28}$$

$$|\mathcal{A}(M_1M_2)M_3\rangle = \frac{1}{\sqrt{2}}(|M_1M_2M_3\rangle - |M_2M_1M_3\rangle)\ , \tag{29}$$

where M_1, M_2 and M_3 are taken to be different meson here. Using this notation, the $SU(3)$ relations take the form

$$\vec{a}^{q}_{K^0K^0\overline{K}^0} = \sum_{f'=\mathcal{S}(M_1M_2)M_3} X_{f'}\vec{b}^{q}_{f'}. \tag{30}$$

Noting that

$$\left\langle K_SK_SK_L|\frac{1}{\sqrt{3}}(\overline{K}^0K^0K^0 + K^0\overline{K}^0K^0 - K^0K^0\overline{K}^0)\right\rangle = \sqrt{\frac{3}{8}}, \tag{31}$$

while the two other orthogonal combinations of $K^0K^0\overline{K}^0$, $K^0\overline{K}^0K^0$ and $\overline{K}^0K^0K^0$ do not contribute, we write

$$\vec{a}^{q}_{SSL} = \frac{1}{\sqrt{8}}(\vec{a}^{q}_{K_0\bar{K}_0K_0} + \vec{a}^{q}_{\bar{K}_0K_0K_0} - \vec{a}^{q}_{K^0K^0\overline{K}^0}), \tag{32}$$

implying

$$\|\vec{a}^q_{SSL}\| \leq \frac{3}{\sqrt{8}} \left\|\vec{a}^q_{K^0K^0\overline{K}^0}\right\| . \tag{33}$$

Using eqs. (15), (16) and (17) with (30) and (33) we obtain

$$\left|\widehat{\overline{\xi}}_{K_SK_SK_L}\right|^2 \leq \frac{9}{8}\left|\frac{V_{us}}{V_{ud}}\right|^2 \frac{\left(\sum_{f'=\mathcal{S}(M_1M_2)M_3} |X_{f'}| \sqrt{\mathcal{B}(B \to M_1M_2M_3)}\right)^2}{\mathcal{B}(B \to K_SK_SK_L)} , \tag{34}$$

where we used the relation $\left\|\vec{b}^q_{\mathcal{S}(M_1M_2)M_3}\right\| \leq \left\|\vec{b}^q_{M_1M_2M_3}\right\|$.

Since there is no measurement of the branching ratio $\mathcal{B}(B \to K_SK_SK_L)$, we cannot put numerical bounds at present. Still, we present the most promising $SU(3)$ relations below.

1. Two $\Delta s = 0$ Amplitudes

We find two amplitude relations involving two $\Delta s = 0$ modes:

$$\vec{a}^q_{K^0K^0\overline{K}^0} = \sqrt{3}\,\vec{b}^q_{\mathcal{S}(\eta_8K^0)\overline{K}^0} - \vec{b}^q_{\mathcal{S}(\pi^0K^0)\overline{K}^0} , \tag{35}$$

$$\vec{a}^q_{K^0K^0\overline{K}^0} = \sqrt{3}\,\vec{b}^q_{\mathcal{S}(\eta_8\overline{K}^0)K^0} - \vec{b}^q_{\mathcal{S}(\pi^0\overline{K}^0)K^0} . \tag{36}$$

The modes $XK^0\overline{K}^0$ with $X \in \{\eta, \pi^0\}$ have not been measured yet (this requires the measurement of both XK_SK_S (XK_LK_L) and XK_SK_L modes).

2. Dynamical assumption

Using the dynamical assumption of section III A 2, we find the relation

$$\vec{a}^q_{K^0K^0\overline{K}^0} = \sqrt{2}\,\vec{b}^q_{\mathcal{S}(\pi^+\overline{K}^0)K^0} . \tag{37}$$

The branching ratio $\mathcal{B}(B \to \pi^+\overline{K}^0K^0)$ is yet to be measured.

C. $B \to K_SK_SK^+$ ($K_LK_LK^+$)

The state $K_SK_SK^+$ can only result from the state

$$\left\langle K_SK_SK^+ | \mathcal{S}(K^0\overline{K}^0)K^+ \right\rangle = \sqrt{\frac{1}{2}} , \tag{38}$$

The orthogonal combination $\mathcal{A}(K^0\overline{K}^0)K^+$ does not contribute. The table of reduced matrix elements is therefore the same table used for $K_SK_SK_L$ above.

A nice feature of charged modes is that the B^+ is a singlet of the U-spin subgroup of $SU(3)$. As a thumb rule, U-spin has a good chance of giving simple relations suitable for our needs since it can change the strangeness of the final state without changing the charge.

Indeed, for $K^0\overline{K}^0K^+$ we find the simple U-spin relation

$$\vec{a}^q_{K^0\overline{K}^0K^+} = \vec{b}^q_{\overline{K}^0K^0\pi^+} . \tag{39}$$

Since we are interested in the symmetric combination, the relation (39) translates to

$$\vec{a}^q_{K_SK_SK^+} = \vec{b}^q_{K_SK_S\pi^+} . \tag{40}$$

The bound on $\left|\widehat{\overline{\xi}}\right|$ is therefore

$$\left|\widehat{\overline{\xi}}_{K_SK_SK^+}\right| = \left|\frac{V_{us}}{V_{ud}}\right| \sqrt{\frac{\Gamma(B^+ \to K_SK_S\pi^+)}{\Gamma(B^+ \to K_SK_SK^+)}} \leq 0.12 , \tag{41}$$

which leads to

$$\left|\overline{\xi}_{K_SK_SK^+}\right| \leq 0.19 . \tag{42}$$

We note that this bound can further improve if the constraint on $K_SK_S\pi^+$ will be strengthend.

D. $B^+ \to K^+K^+K^-$

We have the following relations involving only a single $\Delta s = 0$ amplitude:

$$\vec{a}^q_{K^+K^+K^-} = \sqrt{2}\vec{b}^q_{\mathcal{S}(\pi^+K^+)K^-}, \tag{43}$$

$$\vec{a}^q_{K^+K^+K^-} = \vec{b}^q_{\pi^+\pi^+\pi^-}. \tag{44}$$

Using (44) the bound on $\left|\widehat{\overline{\xi}}\right|$ is therefore

$$\left|\widehat{\overline{\xi}}_{K^+K^+K^-}\right| = \left|\frac{V_{us}}{V_{ud}}\right| \sqrt{\frac{\Gamma(B^+ \to \pi^+\pi^+\pi^-)}{\Gamma(B^+ \to K^+K^+K^-)}} \leq 0.09 , \tag{45}$$

which leads to

$$\left|\overline{\xi}_{K^+K^+K^-}\right| \leq 0.15 . \tag{46}$$

Note that we use the value of $\mathcal{B}(B^+ \to \pi^+\pi^+\pi^-)$ with resonances removed, and that this bound can further improve if the constraint on $\mathcal{B}(B^+ \to \pi^+\pi^+\pi^-)$ will be strengthend.

E. $B^+ \to K_S K_L K^+$

The state $K_S K_L K^+$ can only result from the state

$$\left\langle K_S K_L K^+ | \mathcal{A}(K^0 \overline{K}^0) K^+ \right\rangle = -\sqrt{\frac{1}{2}} \,. \tag{47}$$

The orthogonal combination $\mathcal{S}(K^0\overline{K}^0)K^+$ does not contribute.

There is no measurement yet of the branching ratio $\mathcal{B}(B^+ \to K_S K_L K^+)$. Still, we present the most promising $SU(3)$ relations below.

1. A single $\Delta s = 0$ Amplitude

Since we are interested in the anti-symmetric combination, the relation (39) translates to

$$\vec{a}^q_{K_S K_L K^+} = -\vec{b}^q_{K_S K_L \pi^+} \,. \tag{48}$$

The branching ratio $\mathcal{B}(B^+ \to K_S K_L \pi^+)$ has not been measured yet.

2. Two $\Delta s = 0$ Amplitudes

We also present relations involving two $\Delta s = 0$ modes:

$$\vec{a}^q_{\mathcal{A}(K^0\overline{K}^0)K^+} = \sqrt{\frac{3}{8}}\,\vec{b}^q_{\mathcal{A}(\eta_8\overline{K}^0)K^+} + \sqrt{\frac{1}{8}}\,\vec{b}^q_{\mathcal{A}(\overline{K}^0\pi^0)K^+}, \tag{49}$$

$$\vec{a}^q_{\mathcal{A}(K^0\overline{K}^0)K^+} = \sqrt{2}\,\vec{b}^q_{\mathcal{A}(\overline{K}^0\pi^0)K^+} + \sqrt{3}\,\vec{b}^q_{\mathcal{A}(\pi^0\eta_8)\pi^+}, \tag{50}$$

$$\vec{a}^q_{\mathcal{A}(K^0\overline{K}^0)K^+} = \sqrt{\frac{2}{3}}\,\vec{b}^q_{\mathcal{A}(\eta_8\overline{K}^0)K^+} - \frac{1}{\sqrt{3}}\,\vec{b}^q_{\mathcal{A}(\pi^0\eta_8)\pi^+}. \tag{51}$$

Note that (51) is a linear combination of (50) and (49).

The branching ratios $\mathcal{B}(B^+ \to \eta_8\overline{K}^0 K^+)$ and $\mathcal{B}(B^+ \to \eta_8\pi^0\pi^+)$ have not been measured yet.

IV. CONCLUSIONS

In this work we considered the use of the approximate $SU(3)$ flavour symmetry of the SM to bound the ratio between CKM suppressed and CKM favoured terms in $B \to KKK$

decay amplitudes. This ratio plays an important role in constraining the CP asymmetries of these modes.

We presented several $SU(3)$ relations that can be used to put bounds on the asymmetries. For some $B \to KKK$ modes, the current experimental data is insufficient in order to significantly bound the asymmetries. For those modes, our work can only provide the most promising relations that will allow, once experimental data becomes available, to place stronger bounds on the asymmetries.

For other modes we get the following current constraints:

$$\left|\overline{\xi}_{K^+K^-K^0}\right| \leq \; 0.78, \tag{52}$$

$$\left|\overline{\xi}_{K_SK_SK^+}\right| \leq \; 0.19, \tag{53}$$

$$\left|\overline{\xi}_{K^+K^+K^-}\right| \leq \; 0.15. \tag{54}$$

Future experimental data can lead to stronger bounds. These can be confronted with current and future measured CP asymmetries. Currently, we find all asymmetries to be well within the $SU(3)$ bound.

The work can be extended to include other three body $\Delta s = 1$ modes as well. For example, the measured $B \to K^+\pi^+\pi^-$ or $B \to K^+\pi^-\pi^0$, or states with vector mesons can be considered.

The hope is that, given more and better experimental data, these decay modes and $SU(3)$ relations will provide us with additional unambiguous tests of the SM mechanism of CP violation.

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APPENDIX A: EXPERIMENTAL DATA

We quote experimental data relevant to three pseudoscalar final states. Measurements where resonant contributions are removed from the sample are denoted by (NR). The cur-

rently measured $\Delta s = \pm 1$ modes are [17]:

$$\begin{aligned}
\mathcal{B}(K_S K_S K_S) &= (6.2 \pm 0.9) \times 10^{-6}, \\
\mathcal{B}(K^+\pi^+\pi^-) &= (54.1 \pm 3.1) \times 10^{-6}, \\
\mathcal{B}(K^+\pi^+\pi^-)^{(\mathrm{NR})} &= (2.9^{+1.1}_{-0.9}) \times 10^{-6}, \\
\mathcal{B}(K^+K^-K^+) &= (30.1 \pm 1.9) \times 10^{-6}, \\
\mathcal{B}(K^+K_S K_S) &= (11.5 \pm 1.3) \times 10^{-6}, \\
\mathcal{B}(\eta K^+\pi^-) &= (31.7^{+2.9}_{-3.2}) \times 10^{-6}, \\
\mathcal{B}(K^0\pi^+\pi^-) &= (43.8 \pm 2.9) \times 10^{-6}, \\
\mathcal{B}(K^+\pi^-\pi^0) &= (35.6^{+3.4}_{-3.3}) \times 10^{-6}, \\
\mathcal{B}(K^+\pi^-\pi^0)^{(\mathrm{NR})} &< 4.6 \times 10^{-6}, \\
\mathcal{B}(K^+K^-K^0) &= (24.7 \pm 2.3) \times 10^{-6}, \\
\mathcal{B}(K^0\pi^+\pi^0) &< 66 \times 10^{-6}.
\end{aligned} \tag{A1}$$

The currently measured or constrained $\Delta s = 0$ modes are [17, 21, 23]:

$$\begin{aligned}
\mathcal{B}(\pi^+\pi^-\pi^+) &= 16.2 \pm 1.5, \\
\mathcal{B}(\pi^+\pi^-\pi^+)^{(\mathrm{NR})} &< 4.6 \times 10^{-6}, \\
\mathcal{B}(\pi^+\pi^-\eta) &= (6.2^{+2.0}_{-1.7}) \times 10^{-6}, \\
\mathcal{B}(K^+K^-\pi^+) &< 6.3 \times 10^{-6}, \\
\mathcal{B}(K_S K_S \pi^+) &< 3.2 \times 10^{-6}, \\
\mathcal{B}(K^+\overline{K}^0\pi^0) &< 24 \times 10^{-6}, \\
\mathcal{B}(K^0K^-\pi^+) &< 21.0 \times 10^{-6}, \\
\mathcal{B}(K^+K^-\pi^0) &< 19 \times 10^{-6}, \\
\mathcal{B}(K^+\overline{K}^0\pi^-) &< 18 \times 10^{-6}, \\
\mathcal{B}(\pi^+\pi^-\pi^0)^{(\mathrm{NR})} &< 7.3 \times 10^{-6}.
\end{aligned} \tag{A2}$$

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